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SIMULATION OF THE ENERGY FLOW AT MECHANICAL RESONANCE

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This paper presents the resonance in mechanical systems in aspects of energy states. A single mass mechanical oscillator excited by a rotational mass, set on a cart attached to a spring system was described. A physical model of the oscillator was created and used in experiments. Parameters of the mechanical system were experimentally established on the physical model and then used in simulations. The equation of motion defined by the Lagrangian equation of the second kind was implemented in a Matlab Simulink program. The natural frequency of the mechanical system was calculated and then used as the angular frequency of the rotational mass driven by an electrical gear motor. In both simulation and experimental approaches the mechanical system was excited to the state of resonance. Energies of the cart mass and the rotational mass were compared in simulation and experimental investigations. Additionally the mechanical power supplied by the gear motor was measured and compared to the power of the oscillating mass.

1. Introduction

The vibration amplitude of a mechanical system depends on the ratio of the driving force frequency and the natural frequency of the system [1, 2]. When the excitation force frequency is equal to the natural frequency of the object, the phenomena of mechanical resonance occurs and as a result strong increase in the vibration amplitude can be observed. Single degree of freedom mechanical spring-mass system possesses one natural frequency, which is depending from the spring stiffness and the mass of the object.

Assuming theoretically that no energy dissipation occurs in the mechanical system, in the state of resonance the amplitude of vibration might be consider as infinite. This theoretical simplification cannot find representation in practical approaches due to a certain damping value present in each mechanical system or structure, and thus the amplitude is increased only by a certain finite value. The phenomenon of resonance in mechanical systems is in most cases unwanted. Increased vibration is connected to accelerations and thus to dynamic loads acting on the components of the systems. Overloads and fatigue may strongly influence to the integrity of the construction or structure e.g. Tacoma Bridge, footbridges, shaft critical speeds, etc [3, 4, 5].

The main purpose of this paper is to investigate the mechanisms of energy accumulation in mechanical resonance.

2. Single mass mechanical oscillator

Figure 1 presents the single mass mechanical oscillator. The mass object M is set on a moveable support between two springs. Each spring possess the stiffness coefficient $k/2$ and damping $b/2$. The smaller rotational mass m is used to excite the system.

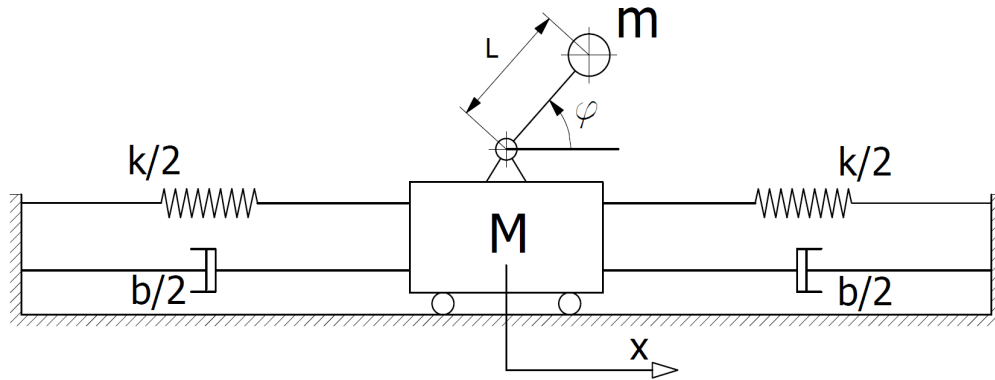


Figure 1. Single mass mechanical oscillator.

The motion of the mechanical system presented in fig. 1 can be described by a mathematical model. In order to define the mathematical model the Lagrangian function is needed, which consists of the difference of kinetic and potential energy of the mechanical system.

$$(1) \quad L = E_k - E_p .$$

The Lagrange equations of the second kind are then defined as:

$$(2) \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) = \frac{\partial L}{\partial q_i} - \frac{\partial D}{\partial \dot{q}_i}$$

where q_i corresponds to the generalized coordinates, L to the Lagrangian function and D to the energy dissipation function.

The number of generalized coordinates is equal to the number of degrees of freedom of the system. In the case of the mechanical system presented in fig. 1, two generalized coordinates should be chosen, although in order to get the system into a resonance state the frequency of the rotating mass m must be set on a constant value equal to the natural frequency of the system. Due to the constant angular velocity one generalized coordinate was chosen, which was the displacement x of the cart.

The kinetic energy of the oscillator can be described as the sum of the cart M and rotating mass m kinetic energy.

$$(3) \quad E_k = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m v_{cx}^2 + \frac{1}{2} I_c \dot{\varphi}^2$$

$$(4) \quad v_{cx}^2 = (\dot{x} + l \dot{\varphi} \sin(\varphi))^2 + \dot{\varphi}^2 l^2 \cos^2(\varphi)$$

$$(5) \quad E_k = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m \cdot [\dot{x}^2 + 2\dot{x}\dot{\varphi} \sin(\varphi) + l^2 \dot{\varphi}^2 \sin^2(\varphi)] + \frac{1}{2} m \dot{\varphi}^2 l^2 \cos^2(\varphi) + I_c \cdot \dot{\varphi}^2$$

The potential energy of the system is equal to the sum of the energy accumulated in the system due to the spring deflection and the energy corresponding to the position change of the rotating mass m .

$$(6) \quad E_p = \frac{1}{2}k \cdot x^2 + m \cdot g \cdot l \cdot \sin(\varphi)$$

After mathematical simplification the Lagrangian function takes the form:

$$(7) \quad L = \frac{1}{2}\dot{x}^2(M + m) + m\dot{\varphi}l \sin(\varphi) + \frac{1}{2}m\dot{\varphi}^2l^2 \underbrace{[\sin^2(\varphi) + \cos^2(\varphi)]}_1 \\ + I_c \cdot \dot{\varphi}^2 - \frac{1}{2}k \cdot x^2 - mgl \sin(\varphi)$$

Energy dissipation function D is related to the damping coefficients included in the mechanical system.

$$(8) \quad D = \frac{1}{2}b\dot{x}^2$$

The equation of motion was defined by solving the equation (2) for the generalized coordinate x.

$$(9) \quad \ddot{x}(M + m) + \ddot{\varphi}ml \sin(\varphi) + \dot{\varphi}^2ml \cos(\varphi) + kx + b\dot{x} = 0$$

Assuming that the rotating mass m has a constant angular velocity, no angular accelerations are present and thus the final form of the Lagrangian equation takes the form:

$$(10) \quad \ddot{x}(M + m) + \dot{\varphi}^2ml \cos(\varphi) + kx + b\dot{x} = 0$$

The rotating mass m will be driven by a small gear motor at a constant frequency equal to the natural frequency of the system. The centrifugal force generated by the rotational mass will then excite the oscillations of mechanical system at resonance.

Equation number 10 does not include the friction forces acting between the cart - rail system. The friction forces acting on the mass M were reduced by use of an industrial low friction cart - rail system, due to that the small friction forces were not included at this stage of investigations. In order to obtain high amplitude oscillations, the damping ratio of the system must be as small as possible.

The energy of the rotating mass m is the sum of the kinetic energy of the mass point m, rotational kinetic energy and the potential energy of the mass m. The damping ratio of the system is reasonably small and for theoretical considerations was not included in the energy description.

$$(11) \quad E_m = \frac{1}{2}mv_{cx}^2 + \frac{1}{2}I_c\dot{\varphi}^2 + mgl \sin(\varphi)$$

The energy of the cart mass M is defined by the oscillation of the mass set on the cart and the potential energy accumulated in the springs.

$$(12) \quad E_M = \frac{1}{2}(M + m)\dot{x}^2 + \frac{1}{2}kx^2$$

3. Simulink model

The equation of motion (10) was used to create a MatLab Simulink model presented in figure 2. The core part of this model is the block diagram representing the equation of motion. A set of gain blocks representing the mass parameters, stiffness and damping coefficients was used to describe the properties of the mechanical system. Integrators were used to calculate the velocity and displacement of the cart.

Ultimately the excitation force takes the form of a sinusoidal function, where the amplitude equals the value of the centrifugal force of the mass m . The "excitation type" subsystem was created in order to provide the possibility to change the excitation signal to step or sweep-sin. Step function was used to simulate the step response of the system. Sweep-sin signal was used together with a simple Matlab script to find the natural frequency of the mechanical system at predefined parameters.

Additional subsystem "Ep & Ek" was created in order to calculate the kinetic and potential energies of the rotational mass m and cart mass M . The planar motion of the mass m defines its kinetic energy as the sum of rotational kinetic energy and kinetic energy of the mass point.

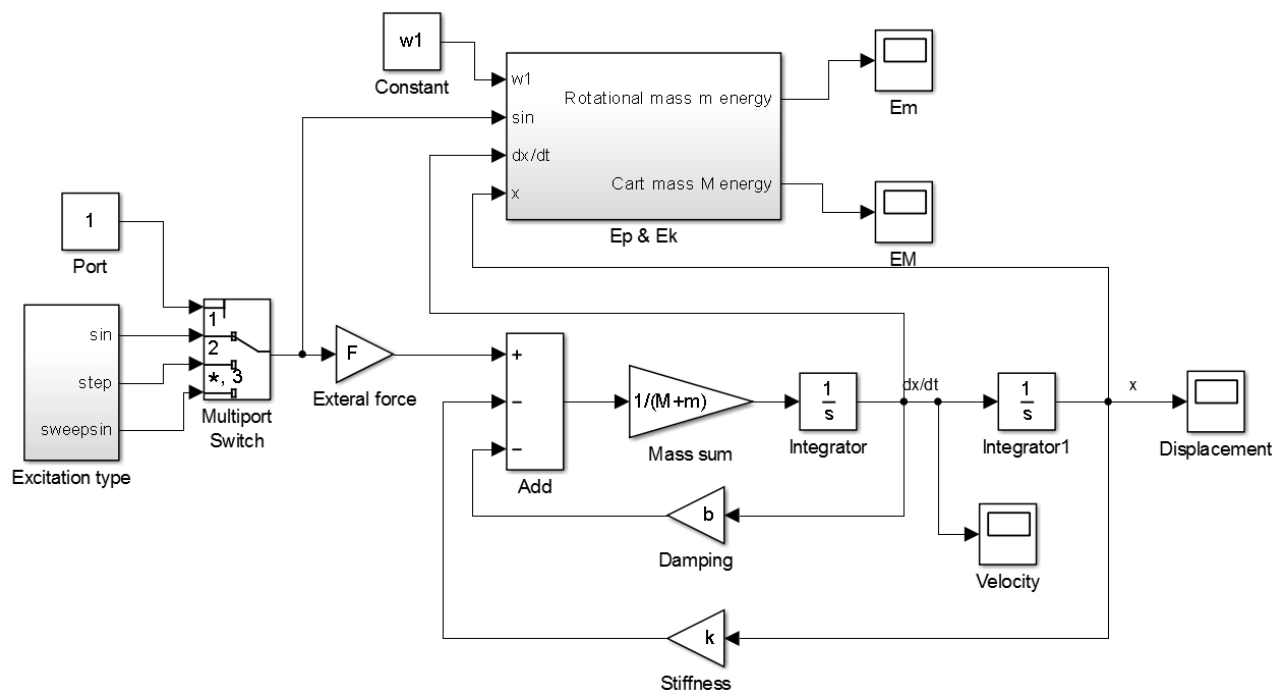


Figure 2. MatLab Simulink block diagram.

The parameters used in simulations were determined experimentally on a physical model of the mechanical system presented in figure 4. The values of parameters were set as:

M - Cart mass	4,554 [kg]
m - Rotational mass	0,593 [kg]
k - Stiffness coefficient	750 [N/m]
b - Damping coefficient	3 [Ns/m]
l - Rotation mass radius	0,108 [m]

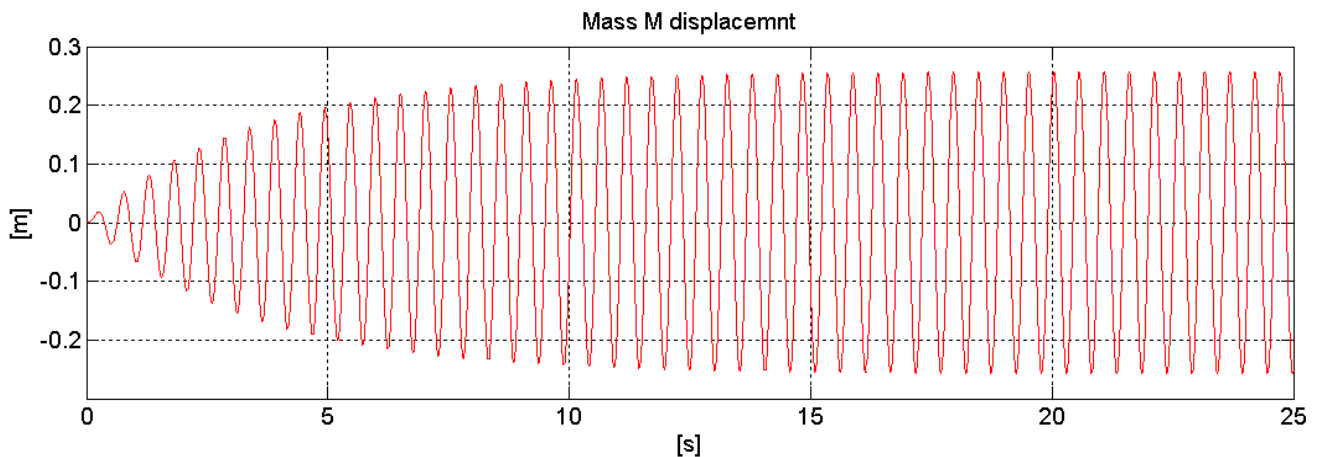


Figure 3. The cart amplitude at resonance - Simulation

Figure 3 presents the oscillation amplitude of the cart at resonance. Simulink model provides also the information on the velocity of the cart. With use of the block diagram presented in figure 2, and the equations (11) and (12) the energy of the pendulum and the energy of the cart were compared. Figure 4 presents the energy of the pendulum in comparison to the energy of the cart mass M at resonance. It can be seen that in this certain theoretical state, the energy of the oscillating mass is about few times higher than the energy of the pendulum.

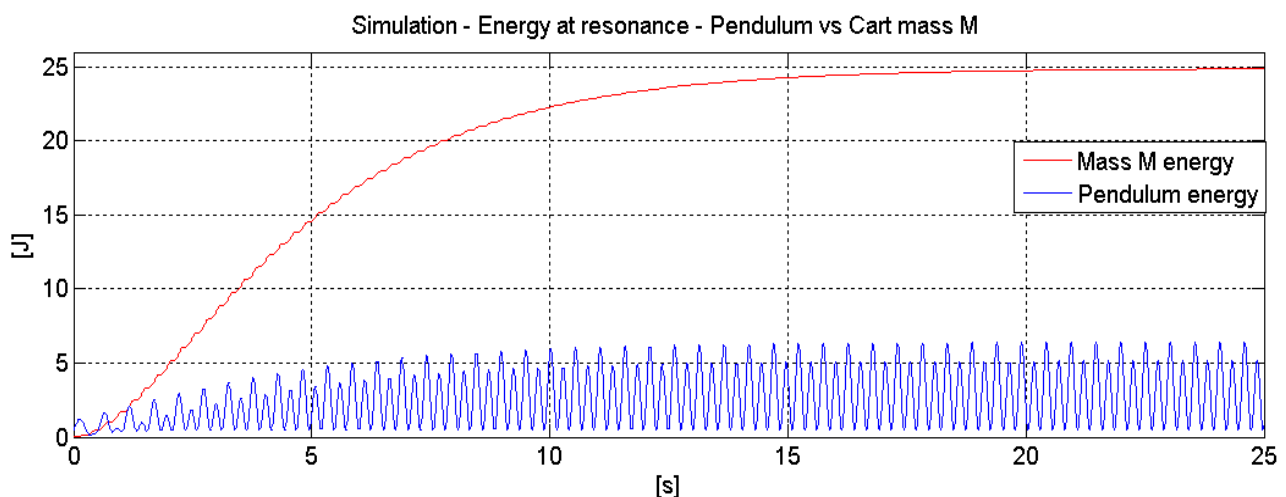
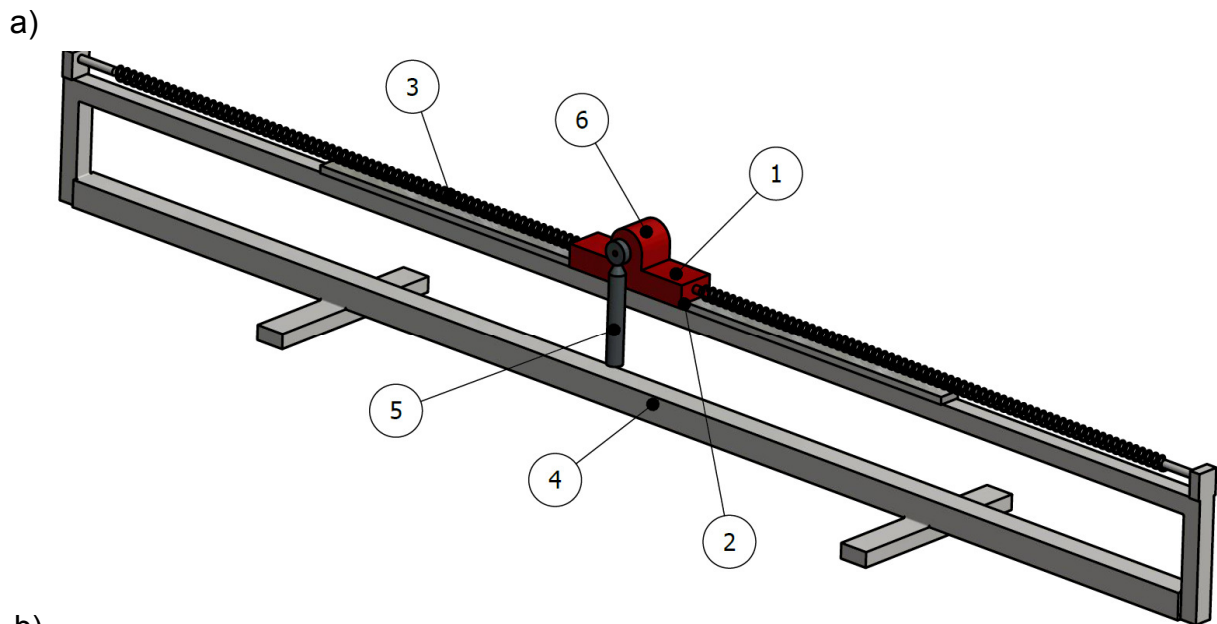


Figure 4. Mechanical energy at resonance- pendulum vs. cart mass M

4. Experimental investigation

Figure 5 presents the single mass oscillator build for experimental investigations. The device consists of the mass M (1) set on a low friction cart-rail system (2). The mass M is connected, from both sides, via a set of two springs (3) to the frame (4) which is fixed to the ground. The external force is generated by the rotation of the pendulum (5), driven by the gear motor (6) set on the mass M. The angular velocity of the motor is controlled by the change of voltage applied to the electrical motor. The oscillation amplitude of the mass M is strongly enhanced as soon as the angular frequency of the pendulum gets closer to the natural frequency of the system.



b)

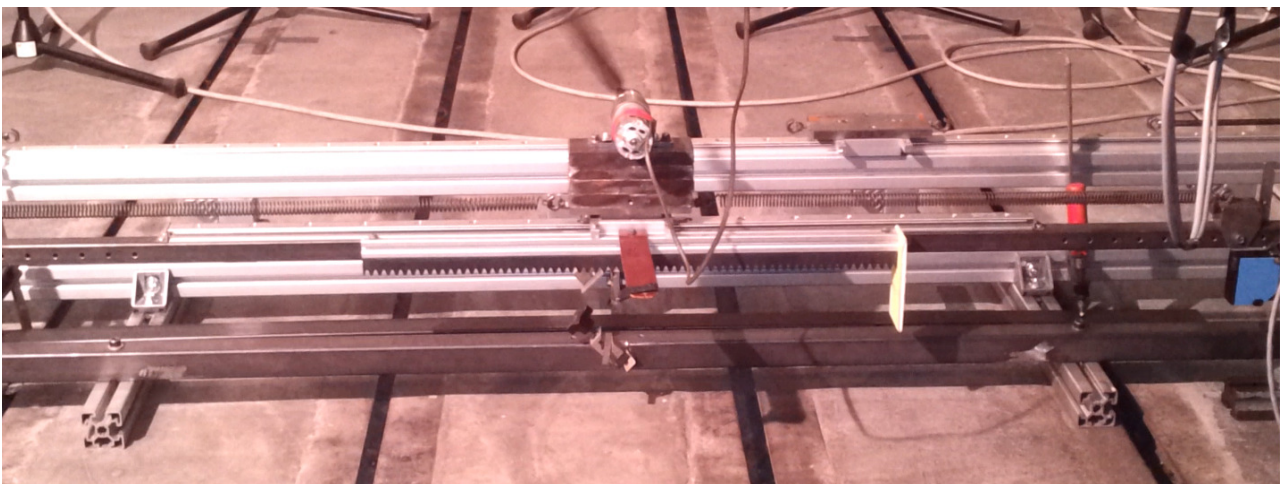


Figure 5. Physical model of a single mass oscillator a) parts description b) view

The rotational mass was driven by an electric gear motor. The angular velocity was controlled manually on a potentiometer of the power supply. The natural frequency was calculated for the SDOF system presented in figure 1. Earlier investigations, where sweep-sin signals were used in Simulink model, were useful in finding the natural frequency of the system. The displacement of the cart was measured with an optical displacement sensor. Figure 5 presents the amplitude of the cart mass oscillations.

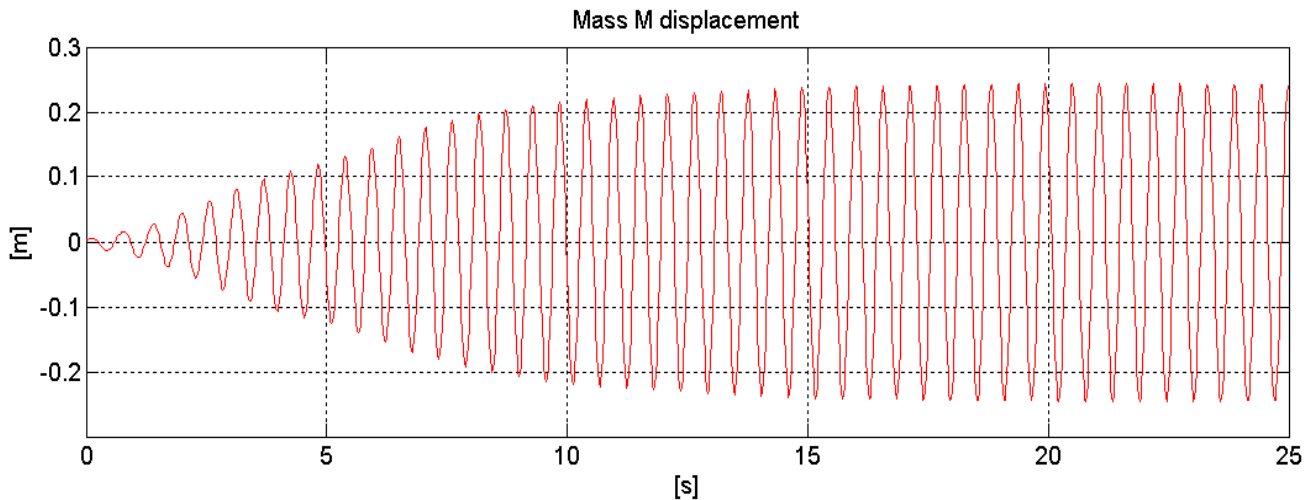


Figure 7. The cart amplitude at resonance - Measurement

The time course of the displacement acquired from measurements, presented in figure 6 is similar to the time course obtained from simulations (figure 3). A difference in the first stage of the displacement can be seen, where the oscillation amplitude is increasing. In case of the physical model, the angular velocity was controlled manually by increasing the voltage supplied to the gear motor. In simulation the frequency of the pendulum was increasing to the natural frequency of the oscillator thus the amplitude growth faster than in the experimental measurements.

The velocity of the cart was calculated as the derivative of displacement signal acquired from measurements. By knowing the velocity of the cart, its displacement, stiffness and mass values the energy of the pendulum and the energy of the cart can be compared. All mathematical operations were carried out with Matlab.

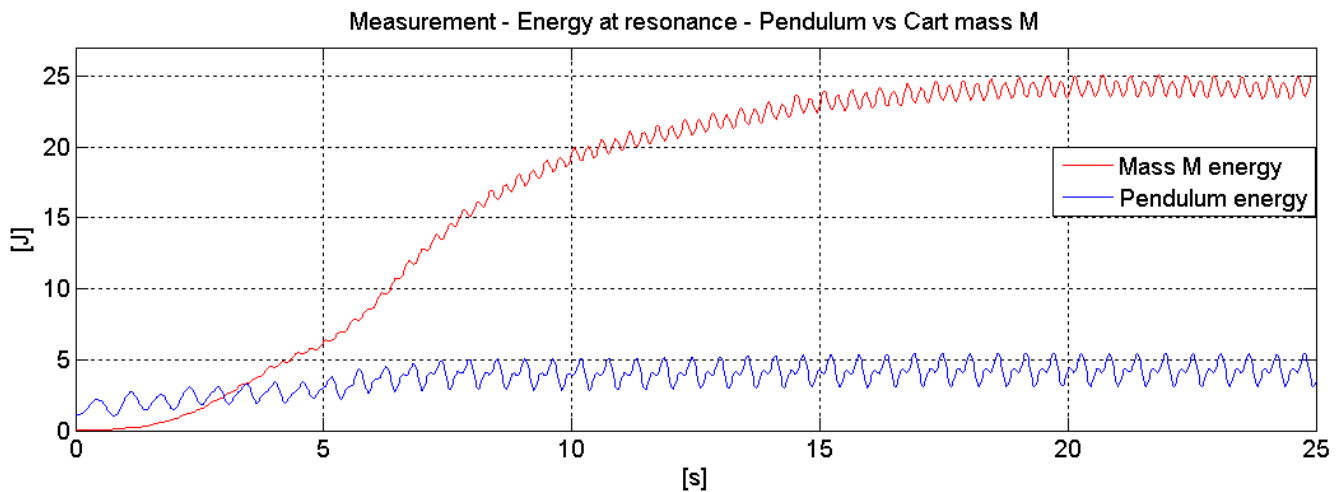


Figure 8. Mechanical energy at resonance - Pendulum vs. Cart mass M - Measurements.

Figure 8 presents the energy of the pendulum in comparison to the cart mass M energy at resonance, acquired in experimental investigations.

To establish the power supplied to the oscillator the measurements of voltage and current on the electrical motor were carried out.

$$(13) \quad P_{EM}(t) = u(t) \cdot i(t)$$

An additional resistor R was used in the electrical circuit of the measurement setup for current measurements [6].

$$(14) \quad i(t) = \frac{u_b(t)}{R}$$

where: $u_b(t)$ - voltage on resistor R

The actual power supplied to the pendulum needs to be reduced accordingly to the efficiency of the small gear motor. Two types of gear motors were chosen for measurements. Both were experimentally tested in case of efficiency in conditions similar, torque and angular velocity, to those occurring at resonance on the oscillator presented in figure 5. The efficiency was found to be 30% and 50% respectively for first and second gear motor. The energy supplied to an oscillator has been established from:

$$(15) \quad E_{IN} = \eta_{EM} \int P_{EM} \cdot dt$$

where: η_{EM} - efficiency of the electric motor

5. Conclusions

Comparing the energy of cart mass M with pendulum mass m energy, both in simulation and experiments, lead to following conclusions. At the steady state of resonance the total energy of the oscillating mass M is several times higher than the energy of the pendulum. This phenomenon can be explained by the ability of the oscillating system to accumulate the energy at resonance and is strongly dependent from the damping. In the resonance conditions the excitation force resulting from the revolution of the pendulum is shifted in phase by 90 degrees to the oscillation of the mass M. The magnification of amplitude is dependent from the cart mass M, stiffness of the spring system and the damping. Further research will be conducted to make the recovery of the amplified energy at the resonance.

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